Lesson: ____
Section: 1.1

Functions & Change

The *idea* of a function: At Taco Bell, the amount of money we spend is a function of the number of tacos we order. The amount of gas we burn is a function of the number of miles we drive.

The word **function** expresses the idea that **knowledge of one fact tells us another.** e.g. If we know the radius of a circle, then circumference is determined. C is a function of r.

If the number of eggs is a function of the number of chickens… what does that mean?

We think of E as a function of C and we call this function f, so we represent this relationship with $E = f(C)$

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- ***Definition* of a function**: A rule that takes certain numbers as inputs and assigns to each *exactly one* definite output number.
- The set of all inputs is called the **domain** of the function. The set of resulting outputs is called the **range**.
- The domain of a function can be explicitly stated or simply implied.
- Sometimes we choose to ***restrict*** the domain.
  For example, in the chicken problem from before, it does not make sense to have a negative number of chickens, so we restrict the domain to values ≥ 0.

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❖ The number of potatoes I need, p
is a function of the number of fries I want, f.
Represent this as a function, q.

❖ Decode and interpret the meaning of the following:

- $30 = q(5000)$
- $q(f + 1000) = p + 10$
- $q^{-1}(50) = 7000$

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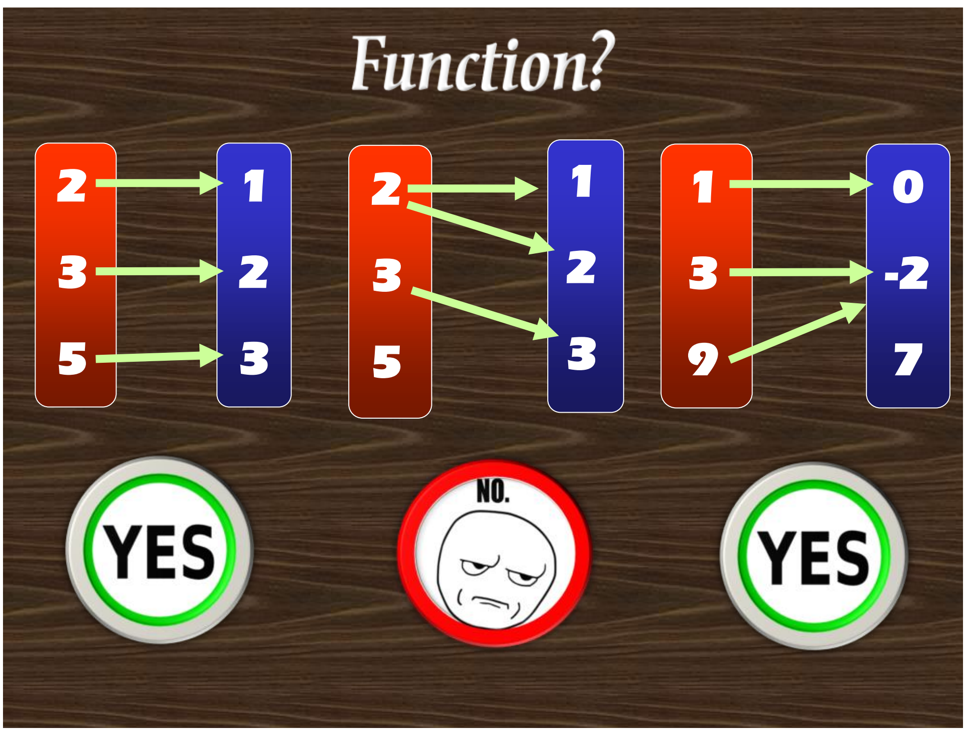

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Independent vs. Dependent

Which variable is independent vs. dependent? Sometimes this is obvious, sometimes it's up to us depending on our point of view. Previously, we used the number of chickens to determine the number of eggs $E = f(C)$, but we could use eggs to find chickens as well $C = g(E)$.

If each output is associated with only one input *and vice-versa*, we call this relationship a **one-to-one function** (the input & output are "married"). The significance of this is that E is a function of C, and C is a function of E, so we can go in either direction easily without any ambiguity. This allows us to define a function ***as well as an inverse*** for that function.

Note that some quantities are **discrete** (only certain values - e.g. dates) while others are **continuous**, which means they can be any number. (e.g. time)

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THE RULE OF FOUR

A relationship between quantities can be represented in many ways. The four most common representations are verbal, numerical, graphical, & analytical.

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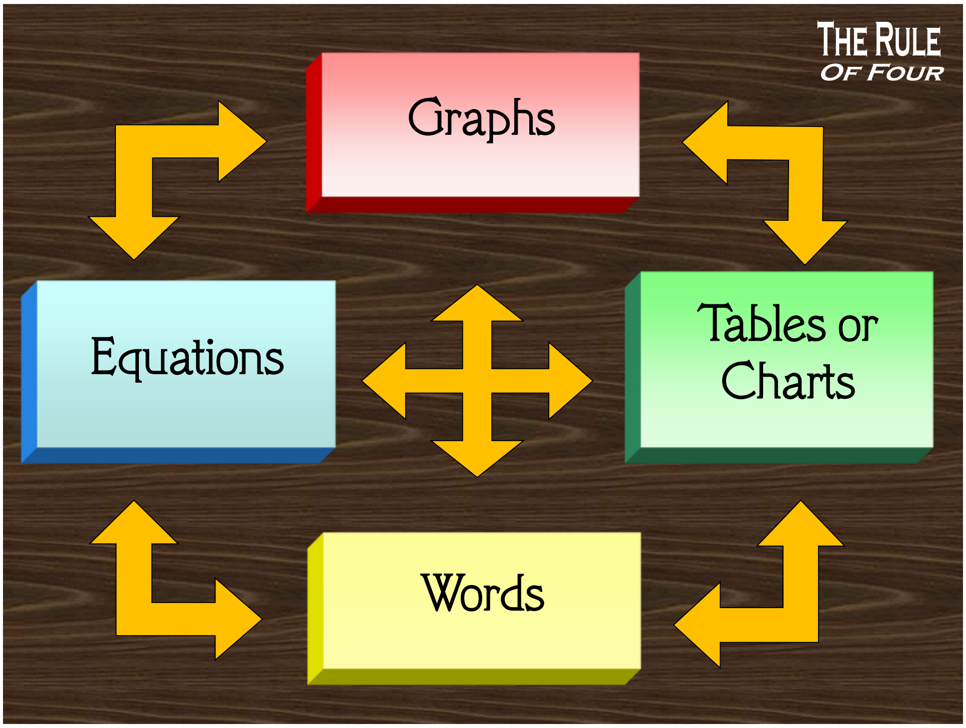
THE RULE OF FOUR
Graphs
Equations
Tables or Charts
Words

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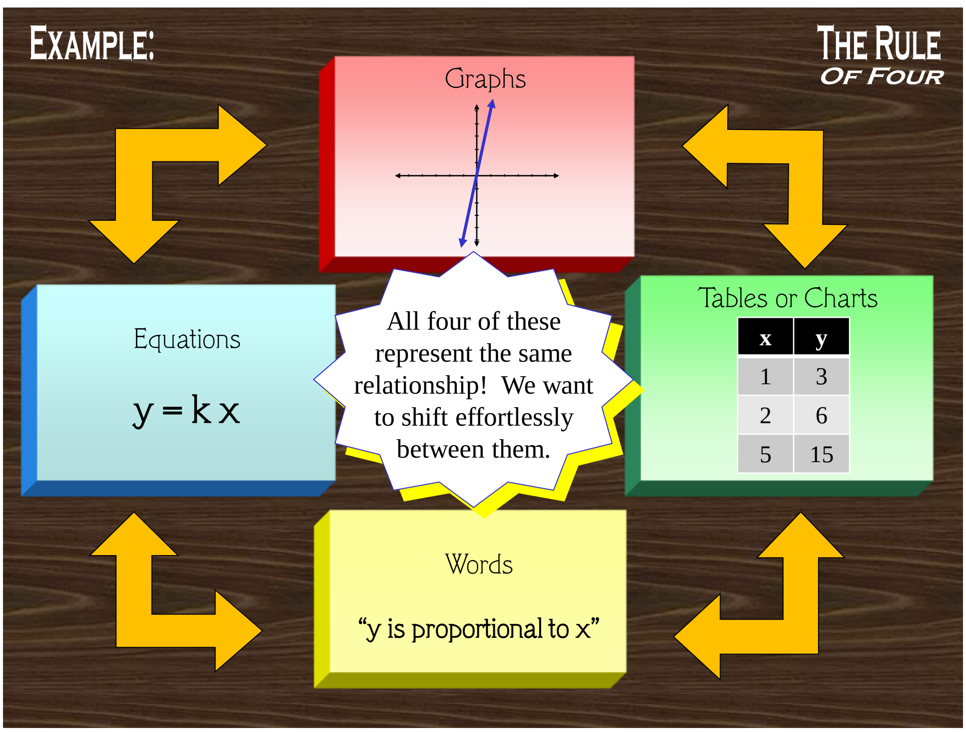
EXAMPLE:
THE RULE OF FOUR
Graphs
Equations
y = k x
All four of these represent the same relationship! We want to shift effortlessly between them.
Tables or Charts
x y
1 3
2 6
5 15
Words
“y is proportional to x”

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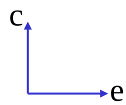
If we say "The number of chickens
is a function of the number of eggs."

We can get our heads around the meaning of this statement using the rule of four! The key is to determine which variable is the ***input*** variable and which is the ***output***.

Graph:



Equation: $c = f(e)$

Numerical: (e, c)

Logical: $e \rightarrow c$

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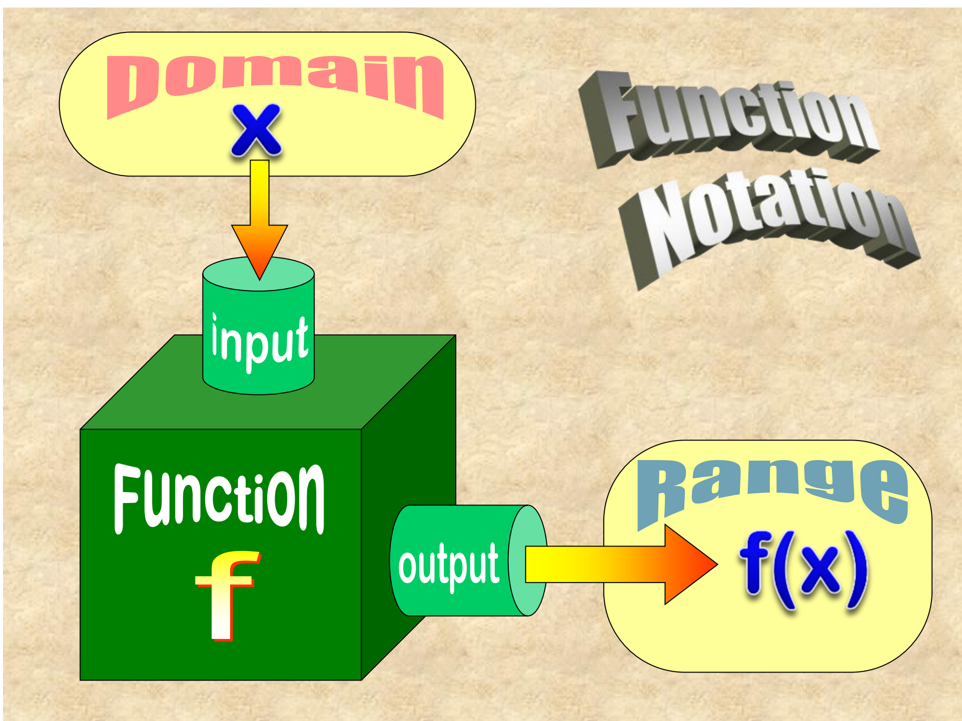

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Function Notation

x ← input

f ← name of the function

f(x) ← output of function *f* at input *x*

What does it mean???

$f(2) = 9$

$f(2x) = 16\,f(x)$

$f(-x) = -f(x)$

$g(x) = f(x) + 7$

If $h(x) = x^2 + 4x$, find $h(3x)$

See Interwrite file for domain & difference quotients – add piecewise

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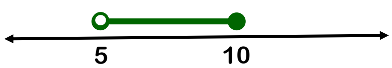
"Intervals" are regions between values (inter vals... between values)

Graphical Notation for Intervals:



Algebraic Notation for Intervals (using Inequalities):

$5 < x \le 10$

Interval Notation:

$(5, 10]$

We say that 5 is an **"open"** endpoint (there is no first value in the interval)

10 is a **"closed"** endpoint (10 is the last value in the interval)

Ex. $(-5, 2] \cap [-2, \infty) = [-2, 2]$

the "*intersection*" of the intervals

Ex. $(-5, 2] \cup [-2, \infty) = (-5, \infty)$

the "*union*" of the intervals

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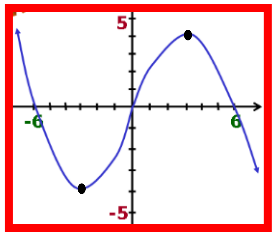
Analyzing the Graph of a Function using Interval Notation

1. Is this a function ?
2. What is the domain? Range?
3. Over what intervals is it Increasing, Decreasing, Constant
4. Where is the function > 0, < 0 ?
5. Where is the slope incr. or decr.?
6. Are there any maxima or minima? Are they relative(local) or global?
7. Is the function even, odd, or neither? Why?



When a graph is ***concave up***, its *slope is increasing*.

When ***concave down***, its *slope is decreasing*

$f(-x) = f(x)$

$f(-x) = -f(x)$

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Difference Quotients

A quotient of 2 differences

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Think: $\frac{\Delta y}{\Delta x}$

Linear Functions

$$y = f(x) = b + mx$$

What are the **constants** in this generic equation? **Variables**?

Formulas like this in which the constants can take on various values give us a **"family" of functions**. These constants (called the **"parameters")** alter the parent function of the family in predictable ways.

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The "Point-Slope Form" of a Linear Equation

with slope m passing throught he point (x_1, y_1)

$$\boxed{y = y_1 + m(x - x_1)}$$

Our favorite form for Calculus!

$$y = y_1 + \frac{\Delta y}{\cancel{\Delta x}}(\cancel{\Delta x})$$

$$y = y_1 + \Delta y$$

This makes sense! y_1 is my *initial value* and Δy is how much the function has *changed* since then.

Ex. Write the equation of a line with a slope of 7 passing through (5,4)

$$y = 4 + 7(x - 5)$$

No need to simplify

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If y is directly proportional to x, then ***y = k x***

If y is inversely proportional to x, then ***y = k ($^1/_x$)***

k is called the "constant of proportionality"

Ex. P = f(g) = 5g

What happens to P if I double the input g?

Ex. M = f(t) = 10 ($^1/_t$)

What happens to M if I double the input t?

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Solving Polynomial Inequalities

Steps:

1. Set inequality to zero
2. Factor if possible
3. Set each factor = 0 to obtain the roots
4. Place the critical number on a ***number line***
5. Test a value in each interval to determine if the function is positive or negative within that interval (These are called **"test intervals"**)
6. Write your solution using interval notation

Ex. $x^2 + 2x < 8$

$x^2 + 2x - 8 < 0$

$(x + 4)(x - 2) < 0$

We need to test to see on which intervals this product is *negative*!

$(x + 4) = 0 \quad (x - 2) = 0$

$x = -4 \quad\quad x = 2$

	$x < -4$	$-4 < x < 2$	$x > 2$
$(x + 4)$	−	+	+
$(x - 2)$	−	−	+
$(x + 4)(x - 2)$	+	−	+

The solution interval is $(-4, 2)$

Now, confirm this algebraic solution by graphing the function and looking to see when the outputs are negative!

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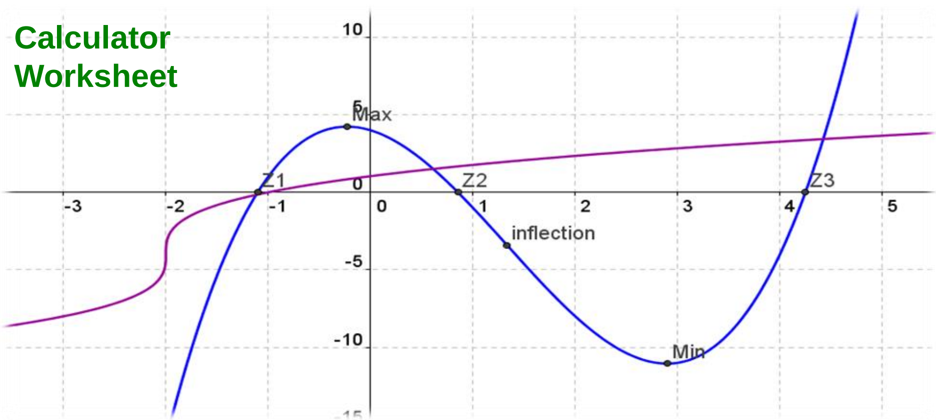
Calculator Worksheet



Graph the functions on your calculator and sketch the graphs on the same set of axes on your paper.

$f(x) = x^3 - 4x^2 - 2x + 4$

$g(x) = 4\sqrt[3]{x + 2} - 4$

1. Over what intervals is f(x)≥0
2. Over what intervals is g(x)>3
3. Over what intervals is f(x) ≥ g(x)
4. Locate all extrema and classify them as relative/absolute
5. Over what intervals is f(x) increasing?
6. Over what intervals is g(x) increasing?
7. Over what intervals is the slope of g(x) increasing?
8. Is f(x) even, odd, or neither?

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