Lesson: ________ Section: **1.1**

Topic: ***Functions & Change***

Lesson: ____
Section: 1.1

Functions & Change

The *idea* of a function: At Taco Bell, the amount of money we spend is a function of the number of tacos we order. The amount of gas we burn is a function of the number of miles we drive.

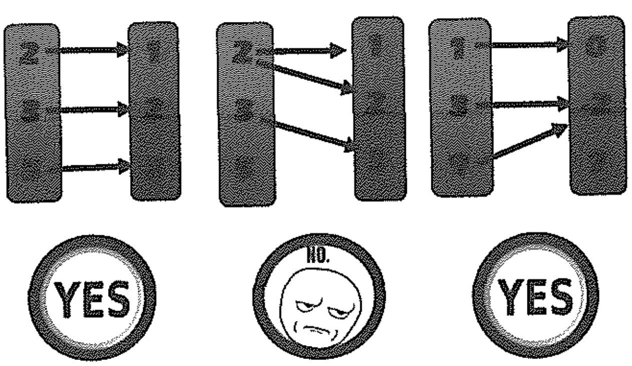
The word **function** expresses the idea that **knowledge of one fact tells us another.** e.g. If we know the radius of a circle, then circumference is determined. C is a function of r.

If the number of eggs is a function of the number of chickens… what does that mean?

We think of E as a function of C and we call this function f, so we represent this relationship with $E = f(C)$

- ***Definition* of a function:** A rule that takes certain numbers as inputs and assigns to each *exactly one* definite output number.
- The set of all inputs is called the **domain** of the function. The set of resulting outputs is called the **range.**
- The domain of a function can be explicitly stated or simply implied.
- Sometimes we choose to ***restrict*** the domain.
  For example, in the chicken problem from before, it does not make sense to have a negative number of chickens, so we restrict the domain to values ≥ 0.

Function?

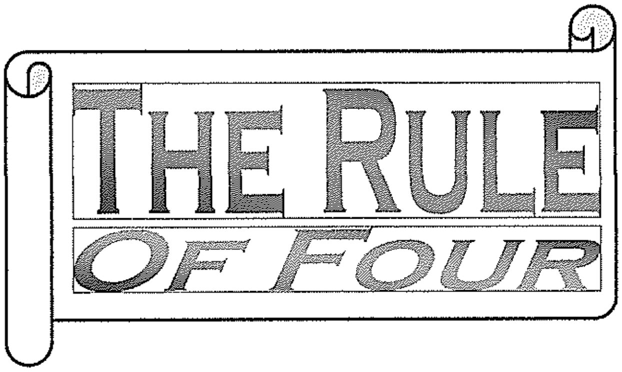


Independent vs. Dependent

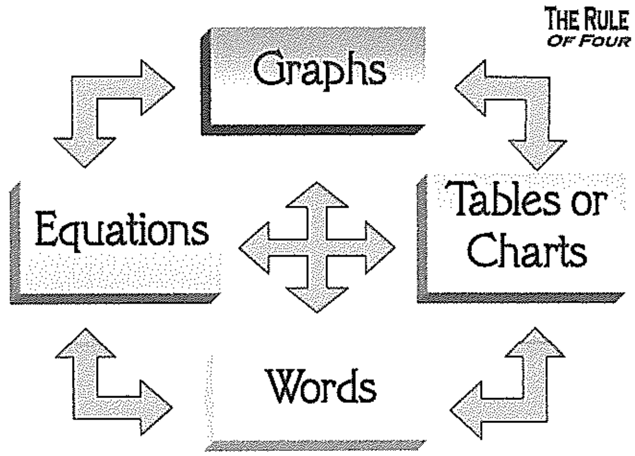
Which variable is independent vs. dependent? Sometimes this is obvious, sometimes it's up to us depending on our point of view. Previously, we used the number of chickens to determine the number of eggs $E = f(C)$, but we could use eggs to find chickens as well $C = g(E)$.

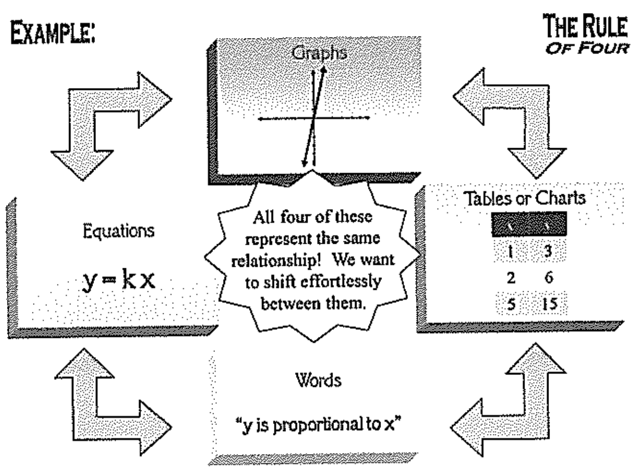
If each output is associated with only one input *and vice-versa*, we call this relationship a **one-to-one function** (the input & output are "married"). The significance of this is that E is a function of C, and C is a function of E, so we can go in either direction easily without any ambiguity. This allows us to define a function ***as well as an inverse*** for that function.

Note that some quantities are **discrete** (only certain values - e.g. dates) while others are **continuous,** which means they can be any number. (e.g. time)



A relationship between quantities can be represented in many ways. The four most common representations are verbal, numerical, graphical, & analytical.

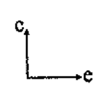




If we say "The number of chickens ***is a function of*** the number of eggs."

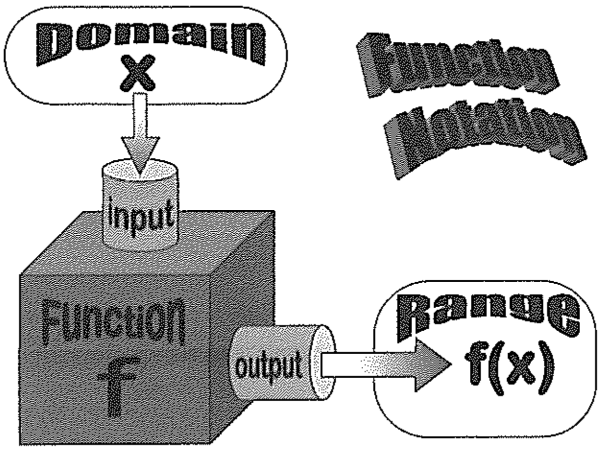
We can get our heads around the meaning of this statement using the rule of four! The key is to determine which variable is the ***input*** variable and which is the ***output***.

Graph:



Equation: $c = f(e)$

Numerical: (e, c)



Function Notation

x ← input

f ← name of the function

f(x) ← output of function *f* at input *x*

What does it mean???

$f(2) = 9$

$f(2x) = 16\,f(x)$

$f(-x) = -f(x)$

$g(x) = f(x) + 7$

If $h(x) = x^2 + 4x$, find $h(3x)$

See Interwrite file for domain & difference quotients – add piecewise

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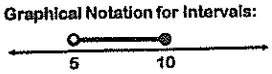
"Intervals" are regions between values (inter vals... between values)

Graphical Notation for Intervals:



Algebraic Notation for Intervals (using inequalities):

$5 < x \leq 10$

Interval Notation:

(5, 10]

We say that 5 is an **"open"** endpoint (there is no first value in the interval)

10 is a **"closed"** endpoint (10 is the last value in the interval)

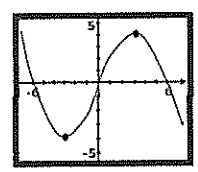
Ex. $(-5, 2] \cap [-2, \infty) = [-2, 2]$

the "*intersection*" of the intervals

Ex. $(-5, 2] \cup [-2, \infty) = (-5, \infty)$

the "*union*" of the intervals

Analyzing the Graph of a Function using Interval Notation



1. Is this a function?
2. What is the domain? Range?
3. Over what intervals is it Increasing, Decreasing, Constant
4. Where is the function > 0, < 0 ?
5. Where is the slope incr. or decr.?
6. Are there any maxima or minima? Are they relative(local) or global?
7. Is the function even, odd, or neither? Why?

When a graph is *concave up*, its *slope is increasing*.

When *concave down*, its *slope is decreasing*

$f(-x) = f(x)$ (even)

$f(-x) = -f(x)$ (odd)

Difference Quotients

A quotient of 2 differences

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Think: $\frac{\Delta y}{\Delta x}$

Linear Functions

$$y = f(x) = b + mx$$

What are the **constants** in this generic equation? **Variables**?

Formulas like this in which the constants can take on various values give us a **"family" of functions**. These constants (called the **"parameters"**) alter the parent function of the family in predictable ways.

The "Point-Slope Form" of a Linear Equation

with slope m passing throught he point (x_1, y_1)

$$\boxed{y = y_1 + m(x - x_1)}$$

Our favorite form for Calculus!

$$y = y_1 + \frac{\Delta y}{\cancel{\Delta x}}(\cancel{\Delta x})$$

$$y = y_1 + \Delta y$$

This makes sense! y_1 is my *initial value* and Δy is how much the function has *changed* since then.

Ex. Write the equation of a line with a slope of 7 passing through (5,4)

$$y = 4 + 7(x - 5)$$

No need to simplify

If y is directly proportional to x, then $y = k\,x$

If y is inversely proportional to x, then $y = k\,(1/x)$

k is called the "constant of proportionality"

Ex. P = f(g) = 5g

What happens to P if I double the input g?

Ex. M = f(t) = 10 ($^1/_t$)

What happens to M if I double the input t?

Solving Polynomial Inequalities

Steps:

1. Set inequality to zero
2. Factor if possible
3. Set each factor = 0 to obtain the roots
4. Place the critical number on a *number line*
5. Test a value in each interval to determine if the function is positive or negative within that interval (These are called **"test intervals"**)
6. Write your solution using interval notation

Ex. $x^2 + 2x < 8$

$x^2 + 2x - 8 < 0$

$(x + 4)(x - 2) < 0$

We need to test to see on which intervals this product is *negative*!

$(x + 4) = 0 \quad (x - 2) = 0$

$x = -4 \qquad x = 2$

		-4	2
$(x+4)$	−	+	+
$(x-2)$	−	−	+
$(x+4)(x-2)$	+	(−)	+

The solution interval is $(-4,2)$

Now, confirm this algebraic solution by graphing the function and looking to see when the outputs are negative!

Lesson: ________ Section: **1.2**

Topic: ***Exponential Functions***

Lesson 1:____ Section 1.2 Exponential Functions

What type of growth or decay is represented in each of the nine situations below?

How do you know?

Notice that the *exponent* is variable

x	f(x)
3	7
4	11
5	15
6	19
7	23

x	g(x)
3	3
4	6
5	12
6	24
7	48

John buys 10 chickens every year for 20 yrs.

Elio increases his egg production by 10% every year for 20 yrs.

$y = 2.4^x$

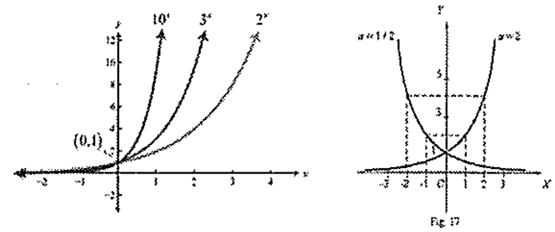
$y = 0.2^x$

$y = e^x$

$y = e^{-x}$

$y = 5e^{0.2x}$

Graphs of the *Family* of *Exponential Functions*



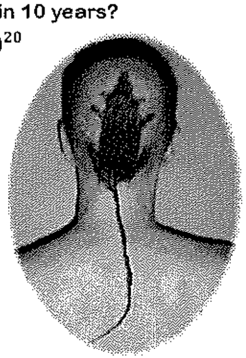
Modeling Exponential Functions

The length of Mr. Gendron's rat's tail increases by 12% every six months. It is currently 2 inches long. How long will it be in 10 years?

$L = 2(1.12)^{20}$

Now let's *generalize*... how long will it be after t years?

$L = 2(1.12)^{2t}$



To solve an exponential word problem...

Ask yourself 3 QUESTIONS

ab^c

a. **What did I start with?**

b. **What number am I multiplying by?**

c. **How many times am I multiplying by this number?**

Flava Flav deposits $10,000 into an account earning 8% annual interest compounded quarterly.
How much will he have after 30 years?

$$B = 10000\left(1 + \frac{0.08}{4}\right)^{4(30)}$$

$$B = \$107{,}651.63$$

Modeling Exponential Functions

Given an exponential function
$y = f(x) = y_0 a^x$
such that $f(20) = 88.2$ and $f(23) = 91.4$, find an equation for y.

$$\begin{cases} 91.4 = y_0 a^{23} \\ 88.2 = y_0 a^{20} \end{cases}$$

Solve the system using the division method

$$\frac{91.4}{88.2} = \frac{y_0 a^{23}}{y_0 a^{20}}$$

$$1.036 = a^3$$
$$1.012 = a$$

Now plug in a to find y_0
$$88.2 = y_0(1.012)^{20}$$
$$69.548 = y_0$$

$$y = 69.548(1.012)^x$$

Exponentials can be converted to any base we choose. Our favorite base in math & science is e ≈ 2.718...

$$P = P_0 e^{kt}$$

This k is a positive value that tells us the *continuous* rate of growth or decay.

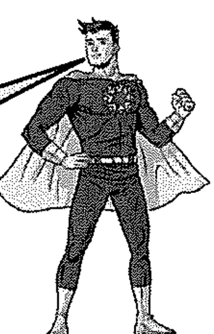
Ex. $P = 20e^{.04t}$ represents an initial quantity of 20 growing at a continuous rate of 4%.

convert $P = e^{0.2t}$ into the form $P = a^t$.

$$a^t = e^{0.2t} = (e^{0.2})^t$$
So $a = e^{0.2} \approx 1.221$

convert $P = 5^t$ to base e.

You'll need to use logarithms for this one you puny humans!



Lesson: ________ Section: **1.3**

Topic: ***New Functions from Old***

Lesson: ____
Section: 1.3

New Functions from Old

A. 1. Transformations
   2. Combining Functions
   3. Odd & Even Functions

B. Inverse Functions

Graphing transformations of functions

Vertical & Horizontal **Shifts** of $y = f(x)$. (any function)

1. Vertical shift c units upward	$h(x) = f(x) + c$
2. Vertical shift c units downward	$h(x) = f(x) - c$
3. Horiz. shift c units to the right	$h(x) = f(x - c)$
4. Horiz. shift c units to the left	$h(x) = f(x + c)$

Reflections of $y = f(x)$ (any function)

1. Reflection in the x-axis	$h(x) = -f(x)$
2. Reflection in the y-axis	$h(x) = f(-x)$

Vertical **Stretches** of $y = f(x)$ (any function)

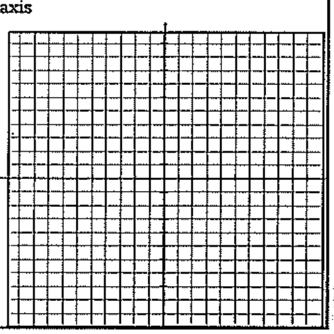
$h(x) = c \bullet f(x)$

"Vertical Stretch by a factor of c"

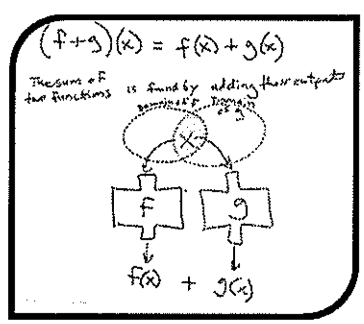
(if $c > 1$) → stretches (if $0 < c < 1$)→compresses

RUN GEOGEBRA
See 2.1 adjustable parabola (if time)

1. $y = \frac{1}{4}x^2$ — 1. Stretch vertically by a factor of 1/2
2. $y = -|x| + 3$ — 1. Reflect over x-axis 2. Shift up 3
3. $y = (-x+2)^3$ — 1. Shift left 2 2. Reflect over the y-axis
4. $y = \sqrt{-x} + 3$ — 1. Reflect over the y-axis 2. Shift up 3
5. $y = 2(|x-2|+3)$
   $= 2|x-2|+6$
   1. Shift right 2
   2. Stretch vertically by a factor of 2. *(multiply all output values by 2)*
   3. Shift up 6



Combining Functions Arithmetically (+, -, x, ÷)



RUN GEOGEBRA
See PC1.5 adding functions

$f(x) = \sqrt{x+4}$

Domain of f: $x \geq -4$

$g(x) = \sqrt{-x}$

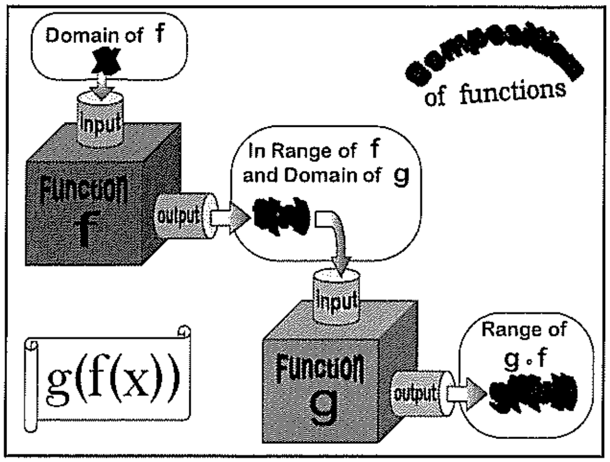
Domain of g: $x \leq 0$

$(f+g)(x) = \sqrt{x+4} + \sqrt{-x}$

Domain of $(f+g)$:

$-4 \leq x \leq 0$

This is the intersection of the domains



Composing Functions

1.2.4

Ex 2) If $f(x)=x^2$ and $g(x)=x+1$, determine:

a) $f(g(2))$ b) $g(f(2))$

c) $f(g(x))$ d) $g(f(x))$

Note: (compare) $f(g(x))$ $g(f(x))$

1.

A composite function has an inner function and an outer function.

(Ex) INNER OUTER

a. $h(t)=(1+t^3)^{27}$

b. $k(x)=\log_2(x^2)$

c. $l(x)=(\log_2 x)^2$

d. $n(y)=e^{-y^2}$

1.2.5

Odd functions and even functions: Symmetry

If a function is symmetric with respect to the y axis then:

a) $f(-x)=f(x)$

b) $f(x)$ is even

Ex: $y=x^2$

$f(-x)$
$=(-x)^2$
$=x^2$
$=f(x)$

$f(-x)=f(x)$

This is an even function.

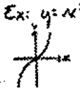
1.1.6

If a function is symmetric with respect to the origin, then:

a) $f(-x)=-f(x)$

b) $f(x)$ is odd

Ex: $y=x^3$



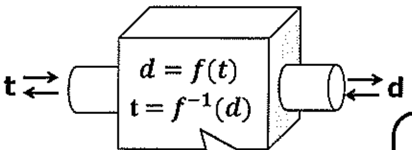
$f(-x)$
$=(-x)^3$
$=-x^3$
$=-(x^3)$
$=-f(x)$

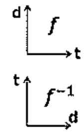
$f(-x)=-f(x)$

This is an odd function.

Note: A function can be even, odd or neither.

Inverse Functions





Note that this is the same relationship just proceeding in the opposite direction. The only difference is which quantity is being considered the output (dependent) and which is the input.
So to describe the inverse of a one to one function, we simply interchange the role of input and output.

Ex. d = 10t

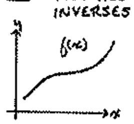
{(1,2), (3,5), (8,7)}

$f(x) = 2x - 5$

$f^{-1}(x) = \frac{x+5}{2}$

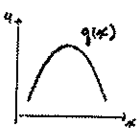
Here the input & output are generic, so we can graph them both on an input vs. output graph.

IV NOT ALL FUNCTIONS HAVE INVERSES



F(x) HAS AN INVERSE SINCE EACH Y VALUE CORRESPONDS TO JUST ONE X.

F(x) IS ONE-TO-ONE



G(x) DOES NOT HAVE AN INVERSE, SINCE A SINGLE Y VALUE CORRESPONDS TO MORE THAN ONE X.

F(x) PASSES THE HORIZONTAL LINE TEST; G(x) DOES NOT.

(13) INVERSE FUNCTIONS

BARRIOS'S RUNNING TIME

d (meters)	f(d) (seconds)
0	0.00
2000	325.90
4000	650.10
6000	975.50
8000	1307.00
10 000	1628.23

DISTANCE RUN BY BARRIOS

t (seconds)	f^{-1}(t) (meters)
0.00	0
325.90	2000
650.10	4000
975.50	6000
1307.00	8000
1628.23	10 000

I. INVERSE FROM A TABLE OF VALUES

TO OBTAIN THE INVERSE, $f^{-1}(x)$ OF A ONE-TO-ONE FUNCTION $f(x)$ FROM A TABLE OF VALUES, INTERCHANGE THE COLUMNS OF THE TABLE.

NOTE: DOMAIN AND RANGE ARE INTERCHANGED WHEN YOU TAKE AN INVERSE.

$f(x) = y_1$ $\quad$ $f^{-1}(x) = y_2$

(x, y) $\quad$ (x, y)

TEMP, CHIRP RATE $\quad$ CHIRP RATE, TEMP

(80, 160) $\quad$ (160, 80)

Note: x represents "input" in a general sense.
This means different things in this problem!

Lesson: ___
Section: 1.4

Logarithms

- Goal: The logarithm as the inverse of the exponential; using the logarithm to solve equations involving exponentials; getting practice using the calculator to solve equations which can't be solved analytically. The base e is just another number; its naturalness as a base for exponentials will become clear in Chapter 3. By varying the constant k in e^{kt}, any exponential can be expressed with base e. The inverse of e^x is the natural logarithm.

Ex₁ Recall our exponential function for the population of Mexico (in millions) since 1980.

$$P(t) = 67.38(1.026)^t$$

Using no calculations,
What is the population of 1980?
What is the percentage growth rate?

When will the population be 200 million?

The logarithm of a number is the exponent to which a base must be raised to produce the given value.

Evaluate:

$\log_2 8 = \mathbf{3}$

$\log_3 \frac{1}{27} = \text{-}3$

$\log_{25} 5 = \mathbf{\frac{1}{2}}$

$\ln e = \mathbf{1}$

$\ln e^7 = \mathbf{7}$

$5^{\log_5 x} = x$

Solve:

$2^x = 11$

$\mathbf{x = \log_2 11}$

$\log_3 x = 5$

$\mathbf{x = 3^5}$

Properties of Logarithms

Change of Base Formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Evaluate $\log_3 7$

Graph with the calculator

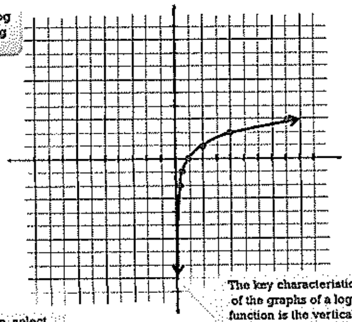
$$y = \log_2 x$$

Graphing $y = log_2 x$ by hand

To understand how to graph the log with base 2, let's start by graphing the exponential with base 2

x	2^x
0	1
1	2
2	4
3	8
-1	½
-2	¼
-3	$^1/_8$

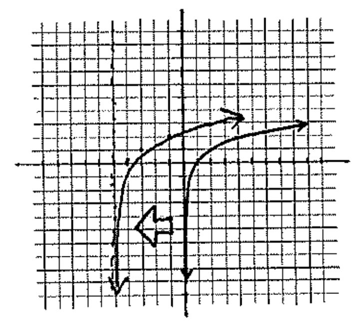
x	$\log_2 x$
1	0
2	1
4	2
8	3
½	-1
¼	-2
$^1/_8$	-3



To graph a log function, select inputs that are powers of your base (e.g. powers of 2)

The key characteristic of the graphs of a log function is the vertical asymptote. This makes sense since exponentials have a horizontal asymptote

Graphing $y = log_2(x+5)$ by hand



Properties of Logarithms

3 Laws of Logarithms

1. $\log_a uv = \log_a u + \log_a v$
2. $\log_a \frac{u}{v} = \log_a u - \log_a v$
3. $\log_a x^n = n\log_a x$

Ex. $\log_2 8x^3$

$\log_2 8 + \log_2 x^3$

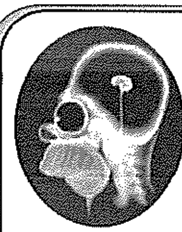
$3 + 3\log_2 x$

Ex. $\log \frac{x}{y\sqrt{z}}$

$\log x - \log\left(yz^{\frac{1}{2}}\right)$

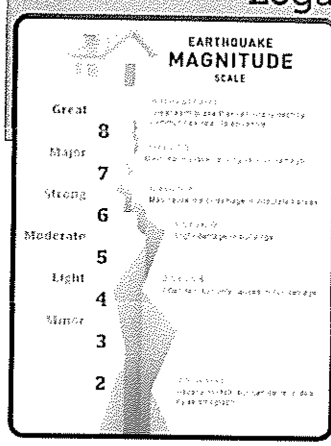
$\log x - \left(\log y + logz^{\frac{1}{2}}\right)$

$\log x - \log y - \frac{1}{2}\log z$



The number of brain cells in your head increases exponentially when you sit in this class. You originally had a measly 10 brain cells, but 3 days later you had 200. How many days would it take for you to have 1700?

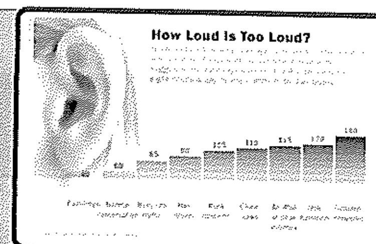
Logarithmic Scales

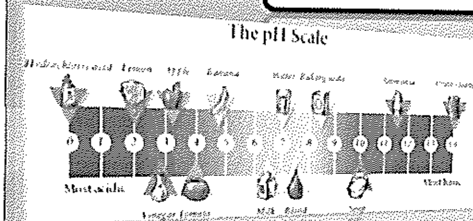


With a logarithmic scale, the numbers represent powers of 10. So comparing an 8 to a 2 is actually comparing 10^8 to 10^2

$$\frac{10^8}{10^2} = 10^6$$

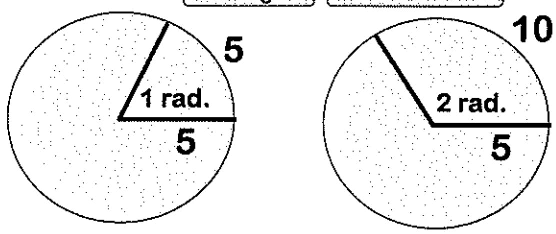
This means an 8 on the Richter Scale is a million times more intense than a 2.





Lesson: _____ Section 1.5
Review of Trigonometry

Triangle | Measurements



The radian measure of a central angle θ $= \dfrac{\text{arc length}}{\text{radius}} = \dfrac{s}{r}$

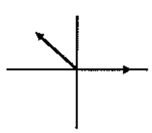
	Radians	Degrees
1 full revolution?	2π	360°
½ a revolution?	π	180°
¼ a revolution?	$\frac{\pi}{2}$	90°
	$\frac{\pi}{3}$	60°
	$\frac{\pi}{4}$	45°
	$\frac{\pi}{6}$	30°
	2	$2\left(\frac{180}{\pi}\right) \approx 114.6°$

$$\theta = \frac{s}{r}$$

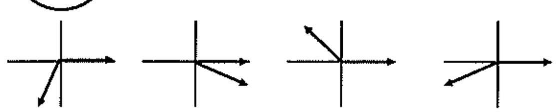
or $s = r\theta$
which is called the arc length formula

Sketch the following angles:

Ex. $\dfrac{3\pi}{4}$

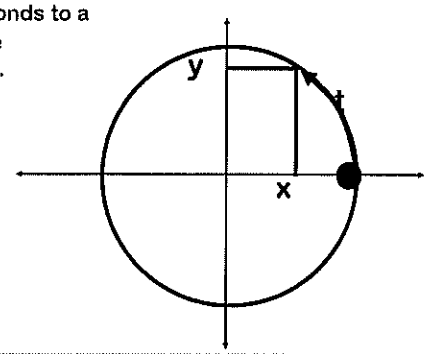


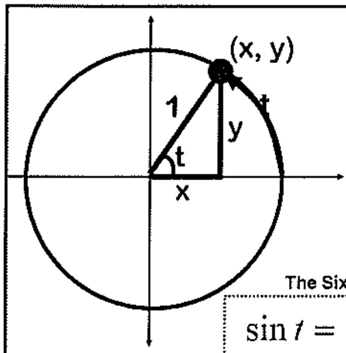
$\dfrac{4\pi}{3}$ $\quad \dfrac{11\pi}{6}$ $\quad \dfrac{-5\pi}{4}$ $\quad \dfrac{19\pi}{6}$

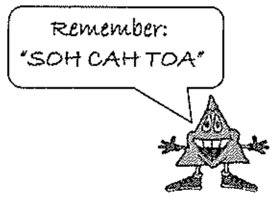


Imagine a real number line is wrapped around the unit circle.

Each real number ***t*** on that line corresponds to a point ***(x,y)*** in the coordinate plane.

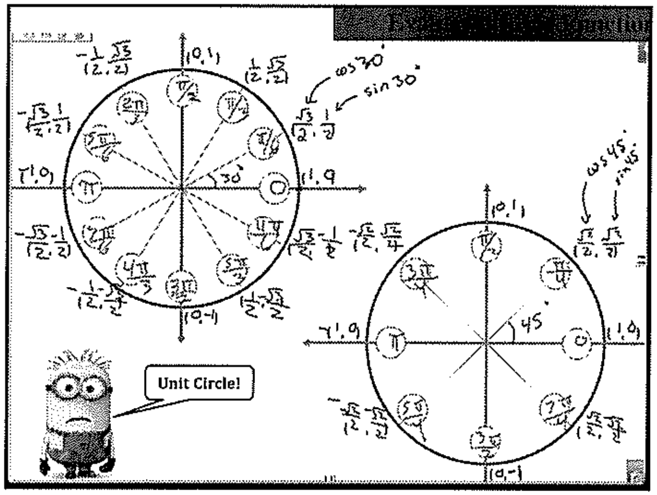






The Six Trigonometric Functions

$\sin t = y$	⟷ Reciprocal Functions	$\csc t = \frac{1}{y}$
$\cos t = x$	⟷	$\sec t = \frac{1}{x}$
$\tan t = \frac{y}{x}$	⟷	$\cot t = \frac{x}{y}$



Memorize!

	30°	60°	45°
sin			
cos			
tan			

ex. $\cos\left(\frac{2\pi}{3}\right)$

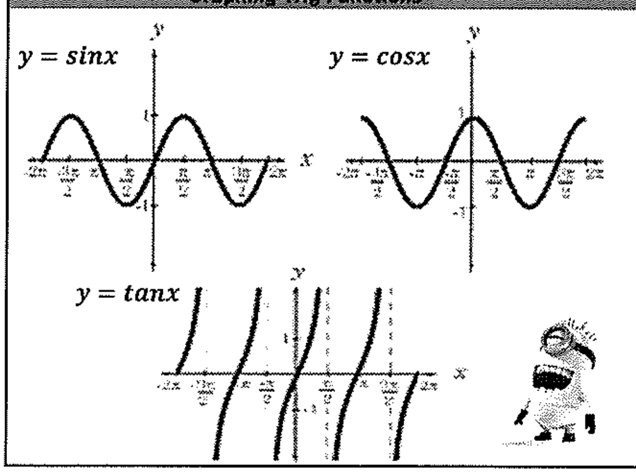
ex. $\sin\left(\frac{11\pi}{6}\right)$

ex. $\tan\left(\frac{5\pi}{4}\right)$

ex. $\sec\left(\frac{\pi}{2}\right)$

ex. $\cos(17\pi)$

Graphing Trig Functions



Graphing Trig Functions

To graph sin and cos: Find the amplitude & the period

period = 2π/b

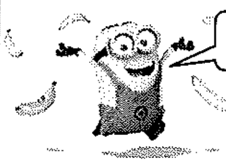
To graph tan and cot: Locate the asymptotes

for tan → set the input = -π/2 and π/2

for cot → set the input = 0 and π

To graph sec and csc: First Graph cos or sin,

then use that graph to locate the asymptotes and max/min points

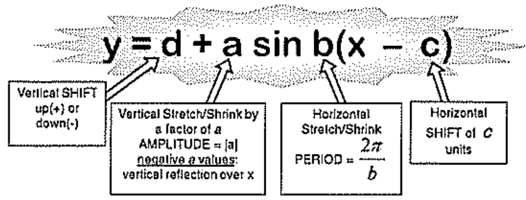


Given a trig graph, how do I organize my work to come up with the equation?

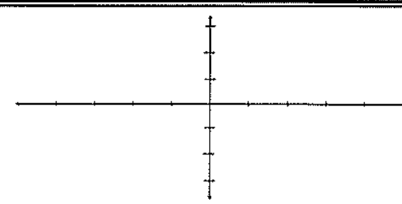
Write down the generic model

y = d + a* sin *b(x-c)

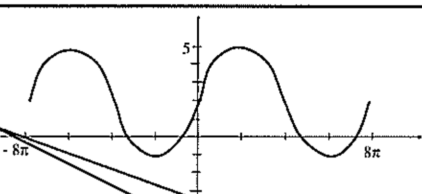
then set up a=__ b= __ c=__ d=__



y = 1 + 2sin 3x



Write the equation of the given function!



y = d + a sin b(x – c)

Write down the generic model then replace all unknown constants with numbers!

a =

b =

c =

d =

What is the angle whose sine is ½ ?

ex$_1$: $\arcsin\frac{1}{2}$

ex$_2$: $\arcsin\frac{\sqrt{2}}{2}$

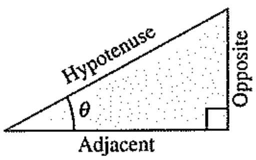
ex$_3$: $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

ex$_4$: $\arcsin 3$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

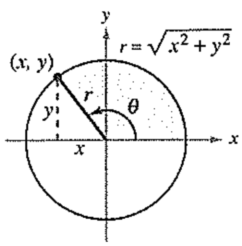


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

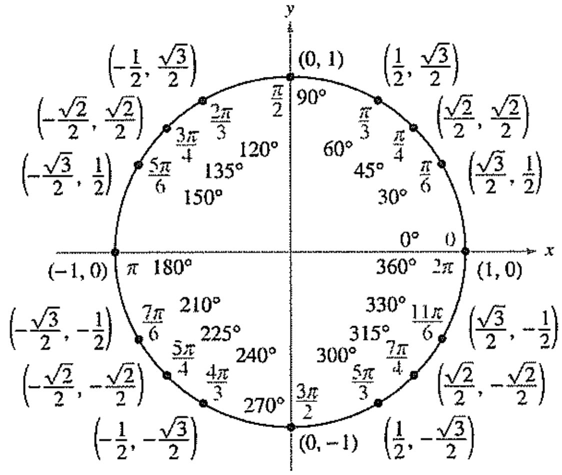
Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

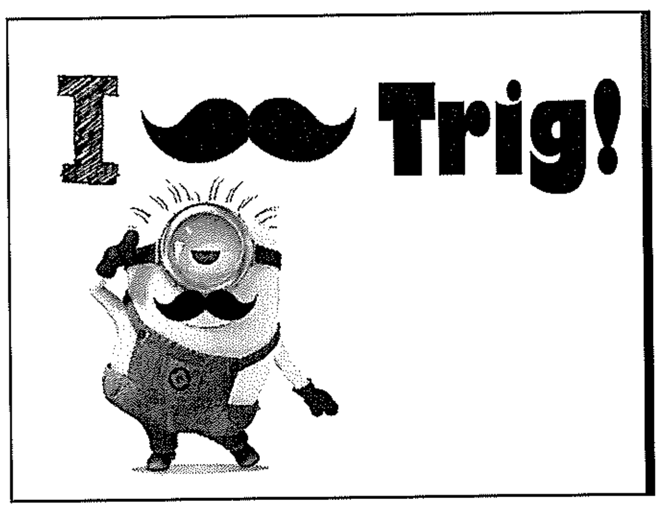
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

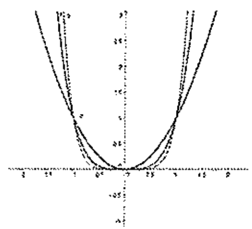
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

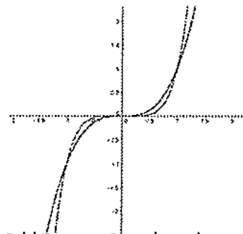


Lesson 1:____ Section 1.6
Powers, Polynomials, & Rational Functions

> A power function has the form
> $f(x) = kx^p$, where k and p are constant.
>
> ex. $y = 2x^3$, $g(x) = 0.2x^4$

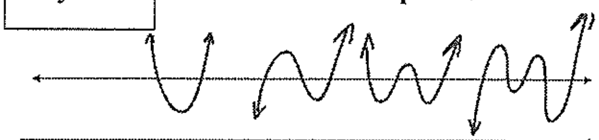


Even Degree : U-shaped



Odd Degree : Seat-shaped

Polynomial Functions are a sum of powers



Degree :	2	3	4	5	n
# Extrema:	at most 1	at most 2	at most 3	at most 4	at most n-1
# x-int :	≤ 2	≤ 3	≤ 4	≤ 5	at most n

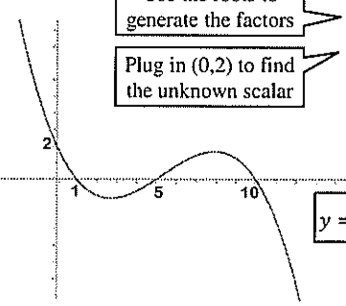
Other Observations?

- The graphs are continuous (pencil never leaves the paper)
- All of the turns are smooth, gradual turns (no sharp corners)
- Domain is all real numbers
- Right & Left behavior are predictable based on lead coefficient and degree

Let's try an example!
Describe 7 features of the graph of
$y = -1/5\, x^3 + 2x^2 - 3x - 4$

1. y-intercept at $(0, -4)$
2. At most 3 x-intercepts (I may be able to find them by factoring using the rational zero test and synthetic division)
3. Falls to the right (Lead Coeff. Test)
4. Rises to the left (since it's an odd degree polynomial)
5. At most 2 turning points
6. Continuous (unbroken graph)
7. Smooth gradual turns (no sharp corners)

Ex. Give a possible equation for the graph shown



Use the roots to generate the factors → $y = k(x-1)(x-5)(x-10)$

Plug in (0,2) to find the unknown scalar → $2 = k(-1)(-5)(-10)$

$2 = k(-50)$

$k = \frac{2}{-50} = -1/25$

$$y = -\frac{1}{25}(x-1)(x-5)(x-10)$$

Rational Functions

$$f(x)=\frac{2x+5}{x^2-2x+3}$$

are a ratio of two polynomial expressions

Understanding Fractions

$$f(x)=\frac{p}{q}$$

What happens to f(x) if p = 0?

What happens to f(x) if q = 0?

GOAL 1 **GRAPHING RATIONAL FUNCTIONS**

CONCEPT SUMMARY — GRAPHS OF RATIONAL FUNCTIONS

Let $p(x)$ and $q(x)$ be polynomials with no common factors other than 1. The graph of the rational function

$$f(x)=\frac{p(x)}{q(x)}=\frac{a_mx^m+a_{m-1}x^{m-1}+\ldots+a_1x+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_1x+b_0}$$

has the following characteristics.

1. x - intercepts are the real zeros of $p(x)$
2. vertical asymptote at each real zero of $q(x)$
3. at most one horizontal asymptote

GOAL 1 **GRAPHING RATIONAL FUNCTIONS**

CONCEPT SUMMARY — GRAPHS OF RATIONAL FUNCTIONS

$$f(x)=\frac{p(x)}{q(x)}=\frac{a_mx^m+a_{m-1}x^{m-1}+\ldots+a_1x+a_0}{b_nx^n+b_{n-1}x^{n-1}+\ldots+b_1x+b_0}$$

3. at most one horizontal asymptote at each zero of $q(x)$

- If $m < n$, the line $y = 0$ is a horizontal asymptote.
- If $m = n$, the line $y=\frac{a_m}{b_n}$ is a horizontal asymptote.
- If $m > n$, the graph has no horizontal asymptote.

DOMINANCE !

$$f(x)=\frac{2x+5}{x^2-2x+3}$$

As x becomes extremely large (x → ∞), who will dominate the fraction, the numerator or the denominator?

As x becomes extremely large (x → ∞), what will f(x) approach?

DOMINANCE !

$$f(x) = \frac{2x^2 + 5}{3x^2 - 4x + 3}$$

As x becomes extremely large (x → ∞), what will f(x) approach?

DOMINANCE !

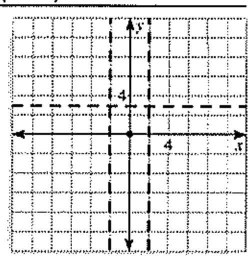
$$f(x) = \frac{x^5 + 3}{x^2 + 4x + 1}$$

As x becomes extremely large (x → ∞), what will f(x) approach?

What happens when x → - ∞?

EXAMPLE **Graphing a Rational Function ($m = n$)**

Graph $y = \frac{3x^2}{x^2 - 4}$.



SOLUTION

The numerator has 0 as its only zero, so the graph has one x-intercept at $(0, 0)$.

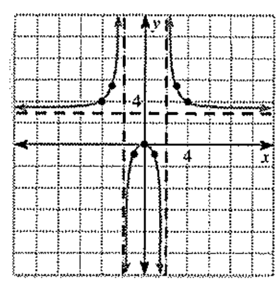
The denominator can be factored as $(x + 2)(x - 2)$, so the denominator has zeros at 2 and -2. This implies vertical asymptotes at $x = -2$ and $x = 2$.

The degree of the numerator (2) is equal to the degree of the denominator (2), so the horizontal asymptote is $y = \frac{a_m}{b_n} = 3$.

EXAMPLE **Graphing a Rational Function ($m = n$)**

Graph $y = \frac{3x^2}{x^2 - 4}$.

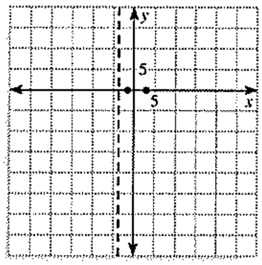
To draw the graph, plot points between and beyond vertical asymptotes.



	x	y
To the left of $x = -2$	-4	4
	-3	5.4
Between $x = -2$ and $x = 2$	-1	-1
	0	0
	1	-1
To the right of $x = 2$	3	5.4
	4	4

EXAMPLE Graphing a Rational Function ($m > n$)

Graph $f(x) = \dfrac{x^2 - 2x - 3}{x + 4}$



SOLUTION

The numerator can be factored as $(x - 3)$ and $(x + 1)$; the x-intercepts are 3 and -1.

The only zero of the denominator is -4, so the only vertical asymptote is $x = -4$.

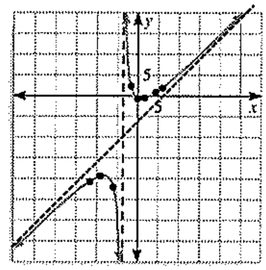
The degree of the numerator (2) is greater than the degree of the denominator (1), so there is no horizontal asymptote and the end behavior of the graph of f is the same as the end behavior of the graph of $y = x^{2-1} = x$.

EXAMPLE Graphing a Rational Function ($m > n$)

Graph $f(x) = \dfrac{x^2 - 2x - 3}{x + 4}$

To draw the graph, plot points to the left and right of the vertical asymptote.

	x	y
To the left of $x = -4$	-12	-20.6
	-9	-19.2
	-6	-22.5
To the right of $x = -4$	-2	2.5
	0	-0.75
	2	-0.5
	4	0.63
	6	2.1



EXAMPLE

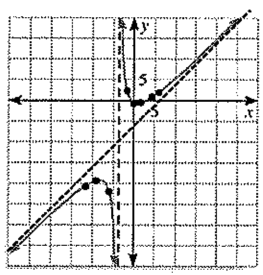
Graph $f(x) = \dfrac{x^2 - 2x - 3}{x + 4}$

We find the slant asymptote using long division!

$$x + 4 \,\overline{)\, x^2 - 2x - 3} \quad = x - 6 + \frac{21}{x+4}$$

$$\begin{array}{r} x^2 - 2x - 3 \\ -(x^2 + 4x) \\ \hline -6x - 3 \\ -(-6x - 24) \\ \hline 21 \end{array}$$

When we are talking about asymptotes, we are talking about "end behavior." As $x \to \infty$ this last term will approach zero, so $f(x)$ will approach the line $y = x - 6$



Confirm with Geogebra (PC-2.7) then explore other rationals and polynomial when viewed "from space."

Sketch the graph of the rational function $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$